Exemplar examination paper

Time: 2 hours **Total: 93 marks**

Instructions

Answer all the questions.

1. Read and understand the questions before you answer.
2. Keep questions and sub-sections of the questions together.
3. Start each question on a new page.
4. Use the same numbering as indicated on the assessment.
5. Leave the margins clear for comments and mark allocation.
6. Rule off after each question.
7. Only use a black or blue ink.
8. Use $\pi = 3{,}142$.
9. All calculations must be approximated to three decimal points.

This assessment task includes a formula sheet.

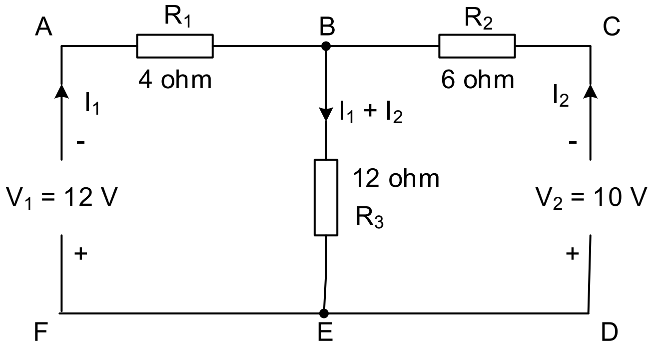
QUESTION 1: KIRCHHOFF'S LAWS

1.1 Make use of an applicable illustration to define the following:

1.1.1 Kirchhoff's Voltage Law (4)

1.1.2 Kirchhoff's Current Law. (4)

1.2 Consider the network below and determine the magnitude of the current through resistor R_3 by making use of Kirchhoff's Laws.

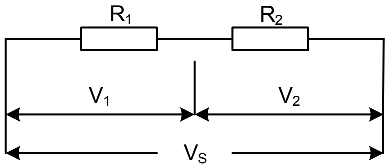


(9)

[**17**]

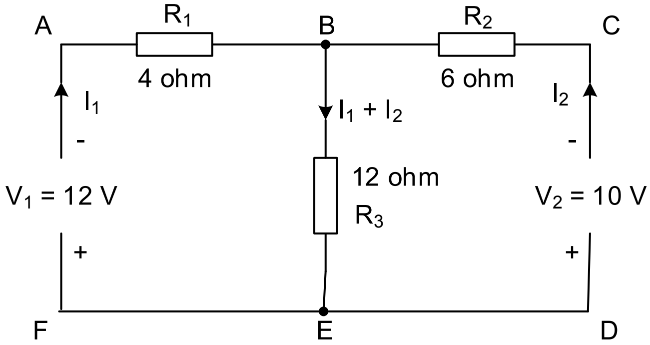
QUESTION 2: SUPERPOSITION THEOREM

2.1 Consider the network below and only give the mathematical expressions that can be used to calculate the voltage across R_1 and R_2.



(4)

2.2 Consider the network below and determine the magnitude of the current through resistor R_3 by making use of the Superposition Theorem.



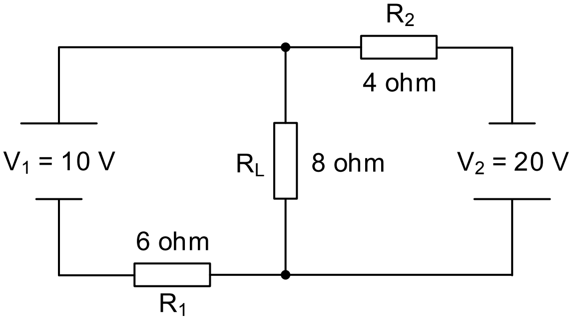
(9)

[13]

QUESTION 3: THEVENIN'S THEOREM

3.1 Define Thevenin's Theorem. (3)

3.2 Consider the network below and determine the magnitude of the current through resistor R_L by making use of Thevenin's Theorem.

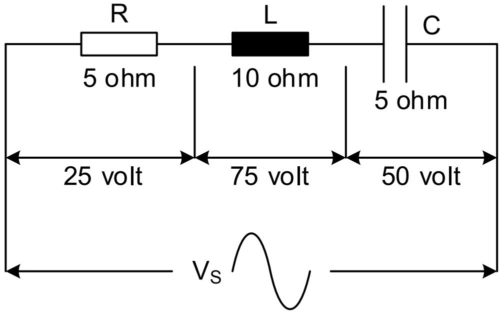


(11)

[14]

QUESTION 4: SERIES RLC NETWORKS

4.1 Consider the network below and determine the following quantities.



4.1.1 The impedance of the network; (3)

4.1.2 The magnitude of the supply voltage; and (3)

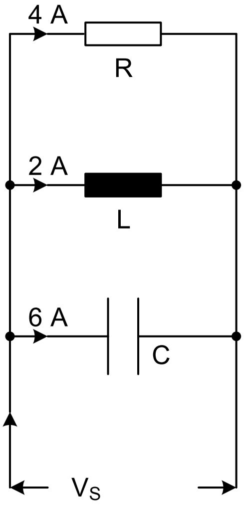
4.1.3 The phase angle between the line current and supply voltage. You are also required to state whether the line current is leading or lagging. Motivate your answer! (4)

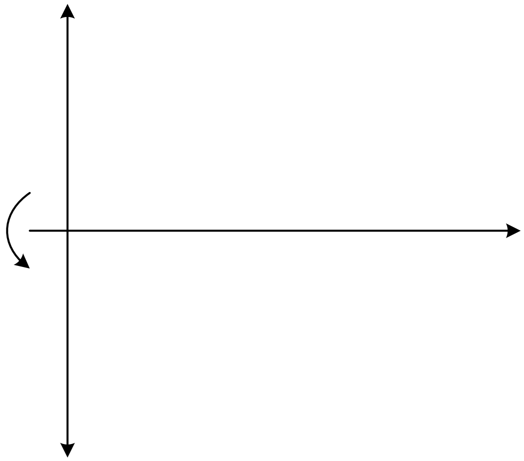
4.2 Draw a neat labelled graphic representation that will illustrate the concept of resonance. (3)

[13]

QUESTION 5: PARALLEL RLC NETWORKS

5.1 Consider figure 5 on the answer sheet and complete the phasor diagram. You are further required to determine the total line current.





(7)

5.2 Draw neat, labelled graphical representations that will illustrate:
  5.2.1 Current versus frequency in a parallel RLC resonant network; and (4)
  5.2.2 Frequency versus impedance in a parallel RLC resonant network (4)

[**15**]

QUESTION 6: Q-FACTOR, BANDWIDTH AND COMPLEX NOTATION

6.1 Give another term for Q-factor. (1)

6.2 Define bandwidth. (3)

6.3 A series RLC network consists of a resistor having a value of 12 ohm, an inductor having a value of 0,15 Henry and a capacitor having a value of 100 µ-Farad connected across a 100 V/50 Hz alternating supply. Use complex notation and calculate the:
  6.3.1 Total impedance of the network; (8)
  6.3.2 Total line current in the network; and (3)
  6.3.3 Voltage drops across the inductor and capacitor. (6)

[**21**]

TOTAL: 93 marks

Formula sheet

Kirchhoff's Laws

$V_R = I_1 \times R$

$P_R = I_2^{\,2} \times R$

Superposition Theorem

Current division

$I_1 = \frac{R_2}{R_1 + R_2} \times I_T$ $\qquad$ $I_2 = \frac{R_1}{R_1 + R_2} \times I_T$

Voltage division

$V_1 = \frac{R_1}{R_1 + R_2} \times V_S$ $\qquad$ $V_2 = \frac{R_2}{R_1 + R_2} \times V_S$

$V_R = I_1 \times R$ $\qquad$ $P_R = I_2^{\,2} \times R$

Thevenin's Theorem

$V_R = I_1 \times R$ $\qquad$ $P_R = I_2^{\,2} \times R$

Norton's Theorem

$V_R = I_1 \times R$ $\qquad$ $P_R = I_2^{\,2} \times R$

Maximum power transfer

$I_L = \frac{V_S}{R_S + R_L}$ $\qquad$ $P_L = I_L^{\,2} \times R_L$

RLC-networks

Resistor

$I = \frac{V_S}{R}$ $\qquad$ $R = \frac{V_S}{I}$ $\qquad$ $V_S = I \times R$

Inductor

$X_L = 2 \times \pi \times f \times L$

$I_L = \frac{V_S}{X_L}$ $\qquad$ $V_L = I_L \times X_L$ $\qquad$ $X_L = \frac{V_S}{I_L}$

Capacitor

$X_C = \frac{1}{2 \times \pi \times f \times C}$

$I_C = \frac{V_S}{X_C}$ $\qquad$ $V_C = I_L \times X_C$ $\qquad$ $X_C = \frac{V_S}{I_C}$

Series RL-network

$Z = \sqrt{R^2 + X_L^{\,2}}$

$V_S = \sqrt{V_R^{\,2} + V_L^{\,2}}$

$V_R = I \times R$ $\qquad$ $V_L = I \times X_L$

$I = \frac{V_S}{Z}$

Series RC-network

$Z = \sqrt{R^2 + X_C^{\,2}}$

$V_S = \sqrt{V_R^{\,2} + V_C^{\,2}}$

$V_R = I \times R$ $\qquad$ $V_C = I \times X_C$

$I = \frac{V_S}{Z}$

$\theta = \tan^{-1} \frac{X_L}{R}$ $\theta = \tan^{-1} \frac{V_L}{V_R}$ $\theta = \tan^{-1} \frac{X_C}{R}$ $\theta = \tan^{-1} \frac{V_C}{V_R}$

Series RLC-network

$Z = \sqrt{R^2 + (X_C \sim X_L)^2}$

$V_S = \sqrt{{V_R}^2 + (V_C \sim V_L)^2}$

$V_C = I \times X_C$ $V_R = I \times R$ $V_L = I \times X_L$

$I = \frac{V_S}{Z}$ $\cos\theta = \frac{R}{Z}$

$\theta = \cos^{-1} \frac{R}{Z}$

True power = $V \times I \cos\theta$ watt

Apparent power = $V \times I$ watt

Parallel RL-network

$Z = \frac{R \times X_L}{\sqrt{R^2 + {X_L}^2}}$ $Z = \frac{V_S}{I_T}$

$I_T = \sqrt{{I_R}^2 + {I_L}^2}$

$I_L = \frac{V_S}{X_L}$ $I_R = \frac{V_S}{R}$

$\theta = \tan^{-1} \frac{I_L}{I_R}$ $\theta = \cos^{-1} \frac{I_R}{I_T}$

Parallel RC-network

$Z = \frac{R \times X_C}{\sqrt{R^2 + {X_C}^2}}$ $Z = \frac{V_S}{I_T}$

$I_T = \sqrt{{I_R}^2 + {I_C}^2}$

$I_L = \frac{V_S}{X_C}$ $I_R = \frac{V_S}{R}$

$\theta = \tan^{-1} \frac{I_C}{I_R}$ $\theta = \cos^{-1} \frac{I_R}{I_T}$

Parallel RLC-network

$Z = \frac{1}{\frac{1}{\sqrt{R^2 + (\frac{1}{X_C} \sim \frac{1}{X_L})^2}}}$ $Z = \frac{V_S}{I_T}$

$I_T = \sqrt{{I_R}^2 + (I_C \sim I_L)^2}$

$I_C = \frac{V_S}{X_C}$ $I_R = \frac{V_S}{R}$

$I_T = \frac{V_S}{Z}$

$\theta = \tan^{-1} \frac{I_X}{I_R}$ $\theta = \cos^{-1} \frac{I_R}{I_T}$

Tuned circuit (tank circuit)

$f = \frac{1}{2 \times \pi \times \sqrt{L \times C}}$

$Z_D = \frac{L}{R \times C}$

$I_L = \frac{V_S}{X_L}$ $I_T = \frac{V_S}{Z}$

$I_L = \frac{V_S}{X_L}$ $I_C = \frac{V_S}{X_C}$

Q-factor, bandwidth and complex notation

$Q = \frac{X_L}{R}$ or $Q = \frac{1}{R}\sqrt{\frac{L}{C}}$

$X_L = j2 \times \pi \times f \times L$

$X_C = \frac{-j1}{2 \times \pi \times f \times C}$

at resonance

$Z = R + j$

$Z = R - j$

$Q = \frac{X_C}{R}$

$Q = \frac{X_L}{R}$

$I = \frac{V}{Z}$

V_L or $V_C = V_S \times Q$

$V_L = I \times X_L$

$V_C = I \times X_C$

$V_R = I \times R$

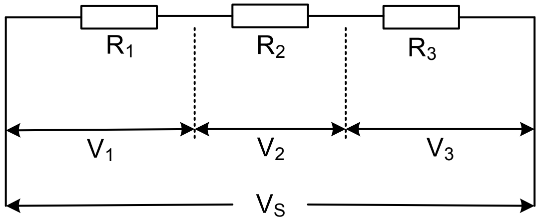
I_L or $I_C = I_T \times Q$

$BW = \frac{R}{2 \times \pi \times L}$ or $BW = \frac{fr}{Q}$

Exemplar examination paper memorandum

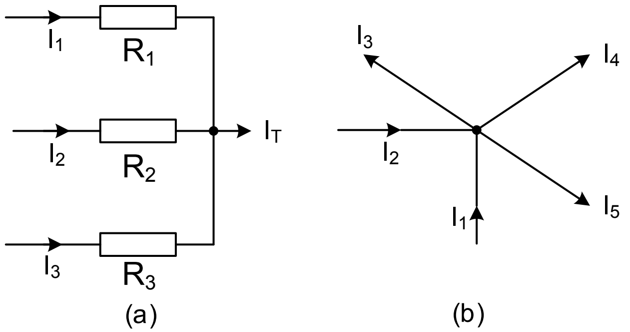
QUESTION 1: KIRCHHOFF'S LAWS

1. 1.1 1.1.1



The algebraic sum ✓ of the individual voltage drops ✓ in a network is equal to the algebraic sum ✓ of the applied voltage. ✓

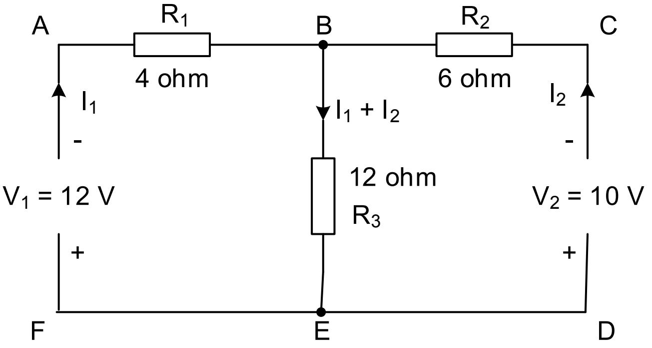
1.1.2



(a) (b)

Kirchhoff's Current Law states that the algebraic sum ✓ of currents entering a point will be equal ✓ to the algebraic sum ✓ of the currents leaving that point. ✓

1.2



Loop ABEFA

$R_1I_1 + R_3(I_1 + I_2) = V_1$

$4I_1 + 12(I_1 + I_2)_1 = 12$

$4I_1 + 12I_1 + 12I_2 = 12$

$16I_1 + 12I_2 = 12$ (1) ✓

Loop CBEDC

$R_2I_2 + R_3(I_1 + I_2) = V_2$

$6I_2 + 12(I_1 + I_2) = 10$

$6I_2 + 12_1 + 12I_2 = 10$

$12I_1 + 18I_2 = 10$ (2) ✓

(1) × 12: $192I_1 + 144I_2 = 144$ (3) ✓

(2) × 16: $\underline{192I_1 + 288I_2} = \underline{160}$ (4) ✓

(3) – (4): $-144I_2 = -16$

$I_2 = 0{,}111\text{ A}$ ✓

Substitute $I_2 = 0{,}111$ A in (1)

$8I_1 + 6I_2 = 12$ ✓

$8I_1 + 6(0{,}111) = 12$ ✓

$8I_1 + 0{,}666 = 12$

$I_1 = 1{,}417\text{ A}$ ✓

$I_{R3} = I_1 + I_2$

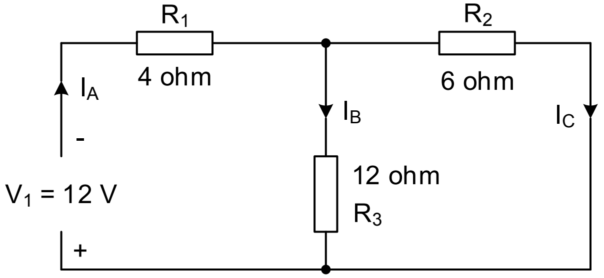
$= 1{,}417 + 0{,}111$

$= 1{,}528\text{ A}$ ✓

QUESTION 2: SUPERPOSITION THEOREM

2. 2.1 $V_1 = \frac{R_1}{R_1 + R_2} \times V_S$ ✓✓ $V_2 = \frac{R_2}{R_1 + R_2} \times V_S$ ✓✓

2.2 Short circuit V_2, indicate current direction and calculate values. ✓



$R_{TA} = R_1 + \frac{R_2 \times R_3}{R_2 + R_3}$

$= 4 + \frac{6 \times 12}{6 + 12}$

$= 8\text{ ohm}$ ✓

$I_A = \frac{V_1}{R_{TA}}$

$= \frac{12}{8}$

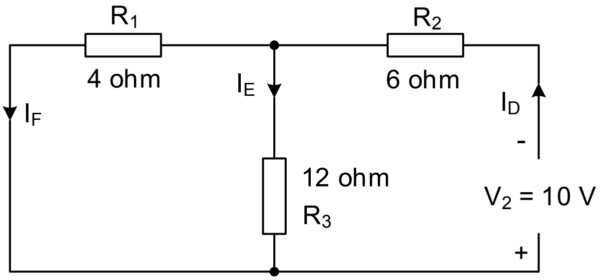
$= 1{,}5\text{ A}$ ✓

$I_B = \frac{R_2}{R_2 + R_3} \times I_A$

$= \frac{6}{6 + 12} \times 1{,}5$

$= 0{,}5\text{ A}$ ✓

Replace V_2 and short circuit V_1, indicate current direction and calculate values. ✓



$$R_{TA} = R_2 + \frac{R_1 \times R_3}{R_1 + R_3} \qquad I_D = \frac{V_2}{R_{TA}}$$

$$= 6 + \frac{4 \times 12}{4 + 12} \qquad = \frac{10}{9}$$

$$= 9 \text{ ohm } ✓ \qquad = 1{,}111 \text{ A } ✓$$

$$I_E = \frac{R_1}{R_1 + R_3} \times I_D$$

$$= \frac{4}{4 + 12} \times 1{,}111$$

$$= 0{,}278 \text{ A } ✓$$

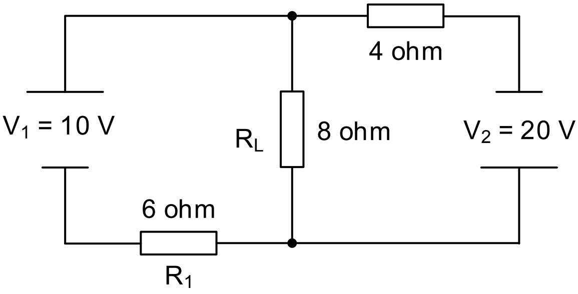
Since I_B and I_E are in the same direction through R_3 they can be added together:

$I_B + I_E = 0{,}5 + 0{,}278 = 0{,}778$ A ✓

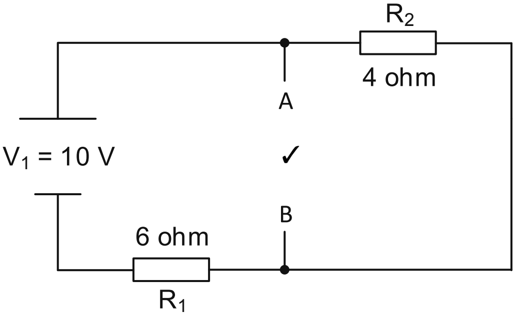
QUESTION 3: THEVENIN'S THEOREM

3. 3.1 A complex network consisting of impedances and voltage sources ✓ may be replaced by a constant voltage source ✓ with series impedance. ✓

3.2



Short circuit V_2, remove R_L and mark terminal A and B. ✓

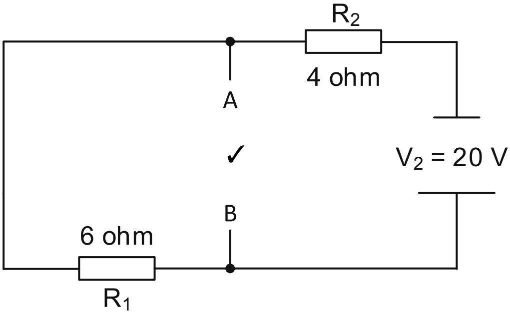


$$V_{TH1} = \frac{R_2}{R_1 + R_2} \times V_1$$

$$= \frac{4}{6+4} \times 10$$

$$= 4 \text{ volt}$$ ✓

Replace V_2, Short circuit V_1. ✓



$$V_{TH2} = \frac{R_1}{R_1 + R_2} \times V_2$$

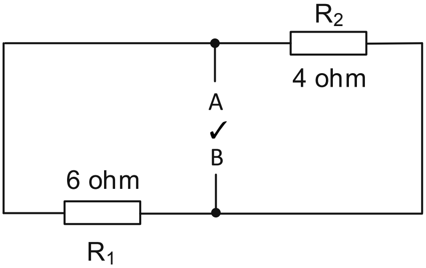
$$= \frac{6}{4+6} \times 20$$

$$= 12 \text{ volt}$$ ✓

$$V_{THT} = V_{TH2} - V_{TH1}$$

$$= 12 - 4$$

$$= 8 \text{ volt}$$ ✓

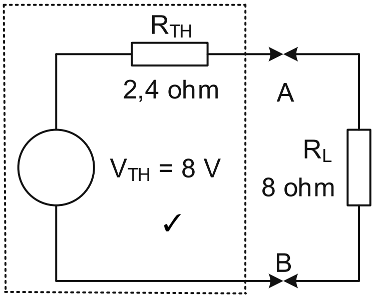


$$R_{TH} = \frac{R_1 \times R_2}{R_1 + R_2}$$

$$= \frac{6 \times 4}{6 + 4}$$

$$= 2{,}4 \text{ ohm} ✓$$

Equivalent network



$$I_{RL} = \frac{V_{THT}}{R_{TH} + R_L}$$

$$= \frac{8}{2{,}4 + 8}$$

$$= 0{,}769 \text{ ampere} ✓$$

QUESTION 4: SERIES RLC NETWORKS

4. 4.1 4.1.1 $Z = \sqrt{R^2 + (X_L - X_C)^2}$ ✓

$= \sqrt{5^2 + (10 - 5)^2}$ ✓

$= 7{,}071$ ohm ✓

4.1.2 $V_S = \sqrt{V_R{}^2 + (V_L - V_C)^2}$ ✓

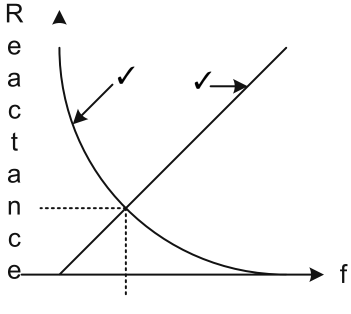
$= \sqrt{25^2 + (75 - 50)^2}$ ✓

$= 35{,}355$ volt ✓

4.1.3 $\theta = \cos^{-1} \frac{R}{Z}$ ✓

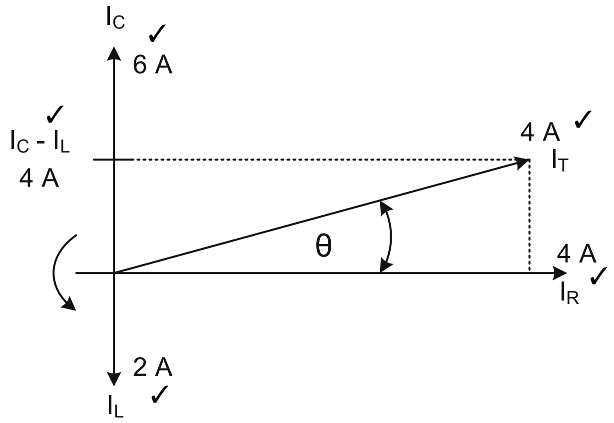
$= \cos^{-1} \frac{5}{7{,}071}$ ✓

$= 44{,}999°$ leading since $V_L > V_C$ ✓



QUESTION 5: PARALLEL RLC NETWORKS

5. 5.1

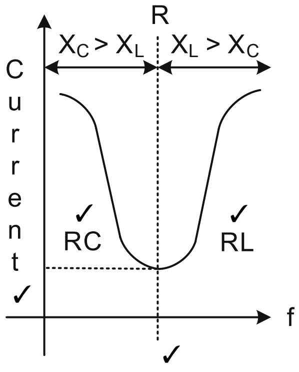


$$I_T = \sqrt{I_R{}^2 + (I_C - I_L)^2} ✓$$

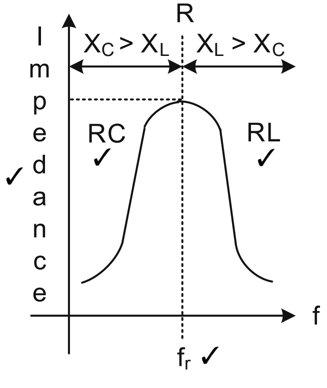
$$= \sqrt{4^2 + (6-2)^2} ✓$$

$$= 4 \text{ ampere} ✓$$

5.2 5.2.1



5.2.2



QUESTION 6: Q-FACTOR, BANDWIDTH AND COMPLEX NOTATION

6. 6.1 Magnification factor ✓

6.2 Bandwidth is that range of frequencies ✓ between two points ✓ where the power has fallen to half of its value. ✓

6.3 6.3.1 $X_L = 2 \times \pi \times f \times L$ ✓

$= 2 \times 3{,}142 \times 50 \times 0{,}15$

$= 47{,}13$ ohm ✓

$X_C = 1$ ✓

$= 2 \times \pi \times f \times C$

$$= \frac{1}{2 \times 3{,}142 \times 50 \times 200 \times 10^{-6}}$$

$= 15{,}913$ ohm ✓

$Z = R + j(X_L - X_C)$ ✓

$= 12 + j(47{,}13 - 15{,}913)$ ✓

$= 12 + j31{,}217$ ohm ✓

$= 33{,}44\underline{|69^\circ}$ ohm ✓

6.3.2 $I_T = \frac{V}{Z}$ ✓

$$= \frac{200\underline{|0^\circ}}{33{,}44\underline{|69^\circ}} \checkmark$$

$= 5{,}98\underline{|-69^\circ}$ ampere ✓

6.3.3 $V_L = I_T \times X_L$ ✓

$= 5{,}98\underline{|-69^\circ} \times 47{,}13\underline{|90^\circ}$ ✓

$= 281{,}737\underline{|21^\circ}$ volt ✓

$V_C = I_T \times X_C$ ✓

$= 5{,}98/\ 69^\circ \times 15{,}913\underline{|-90^\circ}$ ✓

$= 95{,}16\underline{|-159^\circ}$ volt ✓